長庚大學 102 學年度 工學院 / 管理學院 轉學生考試 微積分 試題

考試時間：八十分鐘

答題須知：請詳細閱讀下列試題，並請標明題號依試題順序將答案書寫於答案卷上。

一、填充題 70% (每題 7 分,共 10 題)：請將答案按照序號(1), (2), (3),...作答，不需演算過程。

(1) Find the limit $\lim_{x\to 2} x^{\frac{1}{2-x}}$.

(2)Find the equation of the tangent line to the curve $x^2+4xy+y^2=13$ at the point(2,1).

(3)Find the area of the region bounded by the graphs of

$f(x)=x^3+2x^2-3x,\quad g(x)=x^2+3x$.

(4) Evaluate $\int_1^e x\ln x\,dx$.

(5)Assume that a function f and a number c satisfy

$6+\int_c^x \frac{f(t)}{t^2}dt=2\sqrt{x}$, for all $x>0$. Find $f(x)$.

(6)Find $\int_0^1\int_y^1 \cos(\frac{1}{2}\pi x^2)dxdy$.

(7) Let $f(x,y)=\begin{cases}\frac{x^2y^2}{x^4+y^4} & ,\ (x,y)\neq(0\ ,0)\\ 0 & ,\ (x,y)=(0\ ,0)\end{cases}$, Find $\frac{\partial f}{\partial x}(0,0)$.

(8) Find the area of the region between the inner and outer loops of the curve

$r=1-2\cos\theta$

(9)Find the interval of convergence of the series: $\sum_{k=1}^{\infty}\frac{(-1)^k}{k}x^k$.

(10)Let $f(x,y,z)=x\sin(yz)$. Find the directional derivative of f at (1, 3 ,0) in the direction $u=(1,2,-1)$.

二、計算題 30% (共 2 題)：請務必寫出演算過程，否則不計分。

1.Let $f(x)=\frac{x^2}{x-2}$.

(1) Find all the asymptotes of $y=f(x)$ (找出鉛直或水平或斜漸近線) (5%)

(2) Sketch the graph of $y=f(x)$ (10%)

2.Find the maximum values of $f(x,y,z)=x+2y+3z$

subject to both constraints: $x-y+z=1$ and $x^2+y^2=1$. (15%)