長庚大學九十七學年度研究所碩士班(含在職專班)招生考試試題

所別：機械工程所碩士班　　　　　　　　　　考試科目：工程數學

注意：請詳細閱讀下列試題，並請標明題號依試題順序將答案書寫於答案卷上。

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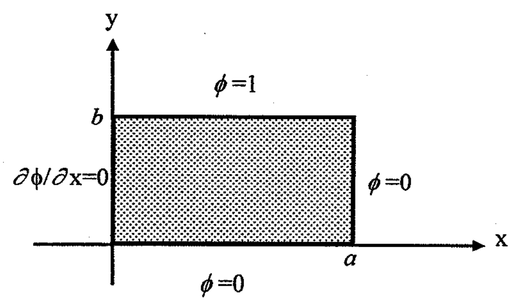
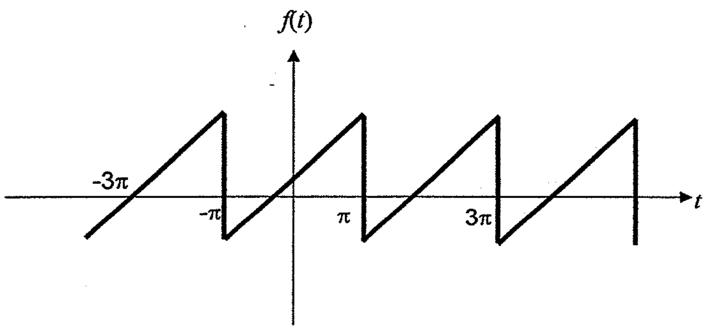
1. (16%) Solve the following ordinary differential equation (O.D.E.), $(D^2+D+1)y(t)=\sin t$, with an initial condition (I.C.), $y(0)=1$, $Dy(0)=0$, where D denotes $\frac{d}{dt}$.

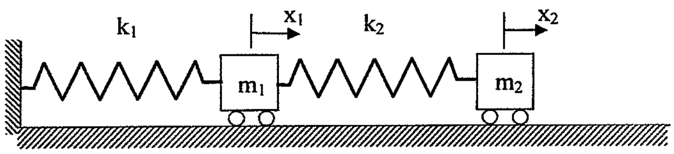
2. (17%) Solve the following partial differential equation, which is a Laplace equation, $\nabla^2\phi = 0$, in a rectangular domain with boundary conditions, $\phi=0$ at $y=0$, $\phi=1$ at $y=b$, and $\frac{\partial\phi}{\partial x}=0$ at $x=0$, and $\phi=0$ at $x=a$. (hint: Laplace Equation is $\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0$)



3. (16%) Find the Fourier series of the periodic sawtooth function,
$f(t)=t+1$ if $-\pi<t<\pi$, and $f(t+2\pi)=f(t)$.



4. (17%) Write the differential equations of a vibration system, as follows. Use matrix form to express the system equations as $\ddot{\mathbf{X}} = \mathbf{AX}$, and calculate the eigenvalues and eigenvectors of matrix **A**, if m_1=2, m_2=1, k_1=4, k_2=2, where $\mathbf{X} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, and $\ddot{\mathbf{X}} = \begin{pmatrix} d^2x_1/dt^2 \\ d^2x_2/dt^2 \end{pmatrix}$. Can you illustrate the natural frequencies and the mode shapes of this vibration system?



5. (18%) Consider that a block of temperature T is placed in a room with a fixed constant room temperature T_r. The temperature of the block is a function of time, $T(t)$. The O.D.E.of T is $\frac{dT}{dt} = -k(T - T_r)$, where k=0.1 and T_r=25°C.

(i) What is the steady-state temperature of the block?

(ii) Solve the problem with an initial temperature, $T(0)$=60 °C.

(iii) Solve the problem with another initial temperature, $T(0)$=0 °C.

Plot the two temperatures versus time, and compare the two solutions.

6. (16%) Using the Laplace transform, solve the following linear O.D.E. with I.C. $y'' + 5y' + 6y = u(t-1) + \delta(t-2), y(0) = 0, y'(0) = 0$,where $u(t)$ is a unit step function, and δ(t) is a delta function. Here $y' = dy/dt$, and $y'' = d^2y/dt^2$.